

* Precession *

Gyroscopes :-

- if the axis of spinning of rotation body is given angular motion about the motion \perp^{e} to the axis of the spin & the ... force acts on the body about the three \perp^{e} axis.
- The torque required to produce acceleration is known as the active gyroscope torque.
- The effect produced by the reactive gyroscope couple is known as the gyroscopic effect.
- It is used to motorcars, aeroplanes and ships etc.

Gyroscopic Angular Velocity :-

The angular velocity of rotary body are.

- 1) The magnitude of the velocity.
- 2) The direction of the axis of rotation.
- 3) The sense of rotation of rotation.

→ The angular velocity has a represented in some vector of manner

* The magnitude of the velocity is represented by the length of the vector.

* The direction of axis of rotation is represented by the drawing the vector \perp^{e} to the axis of rotation (i) normal to the plane of angular velocity.

3) The sense of rotation of rotary is denoted by the taking the direction of vector in a set rules. The general rules are right hands to i.e screw is rotating clock wise (a) anti clock wise.

Gyroscopic angular accel^h

→ let us rotor spin about the horizontal axis ox at the speed of ω rad/sec.

→ Now the magnitude of angular velocity changes to " $\omega + \delta\omega$ "

The spin rotation about ox axis.

→ 'Ac' representing the angular velocity changes in plane normal to AC (i.e.) ox axis.

→ 'cd' representing the angular velocity changes in normal (i.e.) oy axis

→ the change of angular velocity

$$\Delta \omega = (\omega + \delta\omega) \cos \theta - \omega$$

$$\frac{\Delta \omega}{\Delta t} = \frac{(\omega + \delta\omega) \cos \theta - \omega}{\Delta t}$$

$$\frac{\Delta \omega}{\Delta t} \rightarrow 0 \quad \frac{d\omega}{dt} = \frac{(\omega + \delta\omega) - \omega}{dt}$$

$$\frac{d\omega}{dt} = \frac{\delta\omega}{dt} = \frac{d\omega}{dt}$$

The change of angular velocity $c\dot{\theta} = (\omega + \dot{\omega}\theta) \sin\theta$

$$= \left(\frac{\omega + \dot{\omega}\theta}{\delta t} \right) \sin\theta$$

$$= \frac{1}{\delta t} \left(\frac{\omega + \dot{\omega}\theta}{\delta t} \right) \sin\theta$$

Let $\delta t \rightarrow 0$, Then

$$\theta \rightarrow 0 \Rightarrow \sin\theta = \theta$$

$$c\dot{\theta} = \omega\theta + \dot{\omega}\theta$$

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Total accel^h

$$a = x + y axis$$

$$= \frac{d\omega}{dt} + \omega \frac{d\theta}{dt}$$

Gyroscopic torque :-

The moment of inertia of rotor and its angular velocity.

about a horizontal axis of spin ox in the direction, let this

axis of spin turn through a small angle θ in a horizontal

Plane XY to the position ox, in the time period δt .

∴ The change in angular velocity $a_b = \omega \times \theta$

$$\text{angular accel}^h = \omega \times \frac{\delta\theta}{\delta t}$$

$$= \omega \times \frac{d\theta}{dt}$$

angular accel^h is

$$\alpha = \omega \cdot \omega_p$$

$$\text{Torque } T = I \cdot \omega \cdot \omega_p$$

~~Angular velocity in seconds~~ where $k = \text{mass of spin}.$

$$I = m \cdot k^2$$

$$T = m \cdot k^2 \cdot \omega \cdot \omega_p$$

Gyroscopic effects.

- 1) Gyroscopic effects on aeroplanes.
- 2) Gyroscopic effects on naval ships.
- 3) Gyroscopic effects on automobiles.
 - a) motor cycles.
 - b) motor cars.

Gyroscopic effect on aeroplanes :-

The aeroplane in the space, let the propeller is rotating in the clock wise direction, when viewer from the rear end and the angular velocity momentum of are due to the angular velocity.

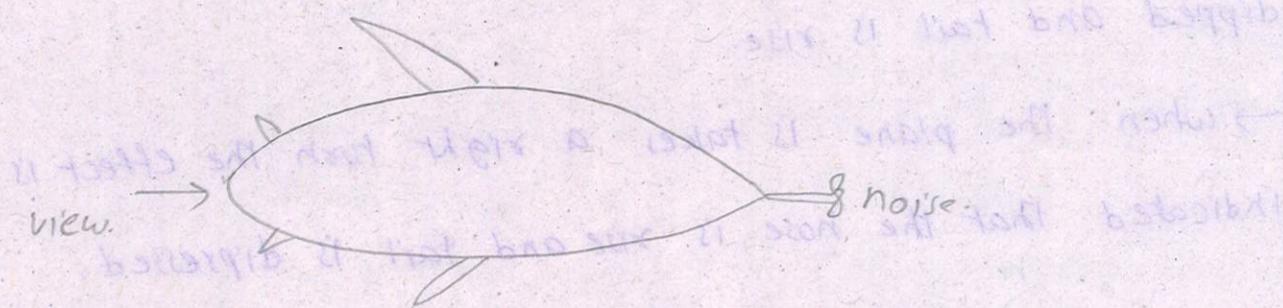
→ If the plane takes left turn, the angular momentum vector shifter and may be represented by the vector.

→ The change in shown in vector ab and is the active gyroscopic couple. This vector is horizontal plane & it's tail to the vector oa' in the limits.

→ The reactive vector is given by $b' \& a'$, which is equal to and opposite to the vector active (ab).

→ The vector shows that the couple acts in the vertical counter clockwise and when view from the right side of the plane.

→ They indicate that it tends to rise the nose & depressed the tail of the aero-plane.



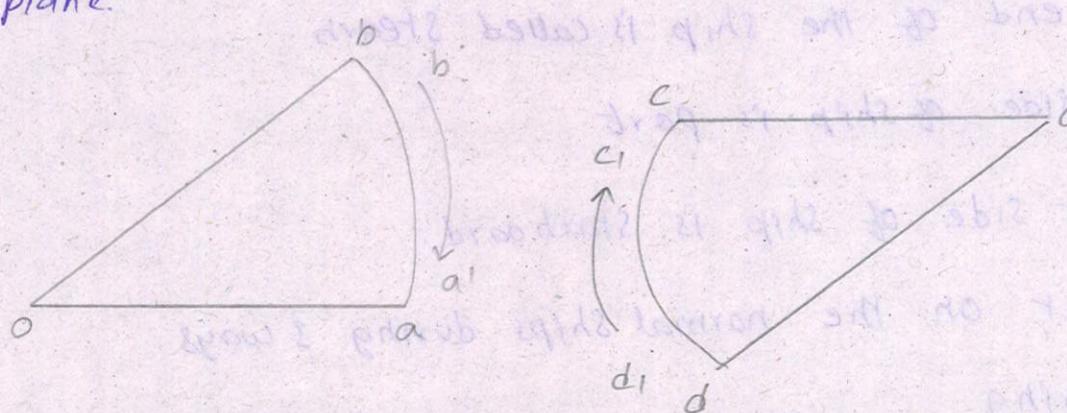
Gyroscopic effect when the aeroplane takes right turn.

The change in vector shown by cd and is the active gyroscopic couple.

→ It is \perp to the vector oc , the limits in the horizontal plane, the reactive couple is $c' \& b'$.

→ The couple acts on the vertical plane and is clockwise when viewed from right hand side of the plane.

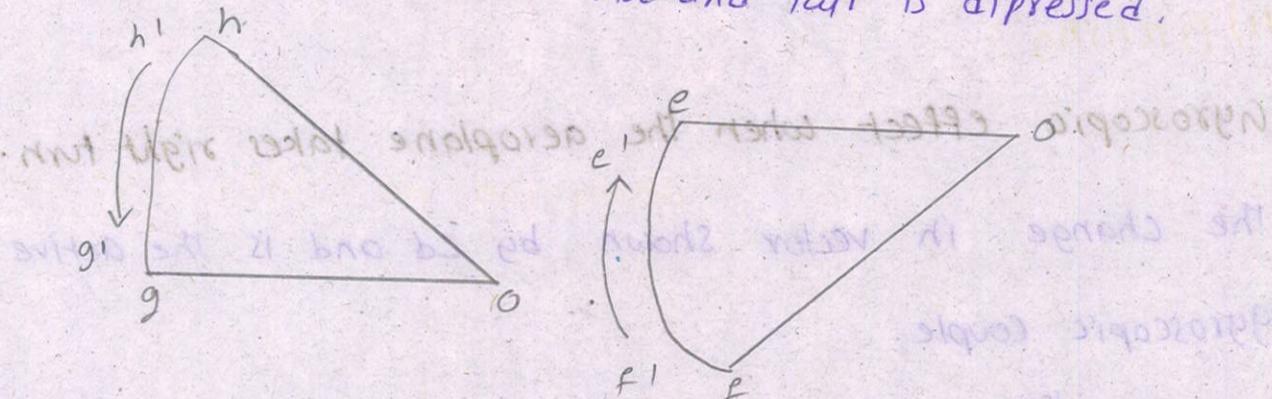
→ Thus, it tends to dip the nose and rise the tail to the aeroplane.



→ If the rotation of engine is reversed i.e. it rotates counter clockwise, when viewing from the rear end the angular momentum vector due on taking a left turn it changes to of the active gyroscopic effect and it changes.

→ Viewing from the right hand of the plane, the nose is dipped and tail is rise.

→ When the plane is takes a right turn the effect is indicated that the nose is rise and tail is depressed.



where $C = \text{gyroscopic couple}$.

$\omega_p = \text{angular acceleration}$

Gyroscopic effect on ships:-

- 1) The fore head of the ship are called bow.
- 2) rear end of the ship is called stern.
- 3) left side of ship is port.
- 4) right side of ship is starboard.

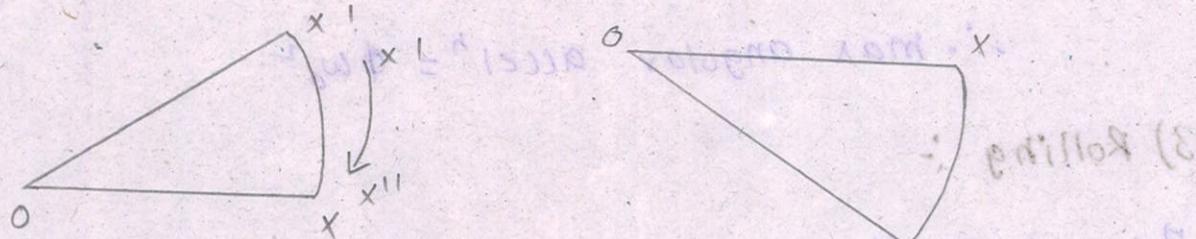
→ Effect on the normal ships during 3 ways:

- 1) Steering
- 2) Rolling & Pitching.

i) Steering :

it is the turning of a complete ship in a curve toward left or right if moves forward.

→ consider a ship taking a left turn the rotor clockwise direction.



ii) Pitching :

→ it is a moment of the ship up and down in vertical like about Transfer axis, the Transfer axis and the axis of Precession and the pitching of the ship is assumed to take place with the simple harmonic motion.

$$\text{Angular Velocity of STM} = \frac{2\pi}{\text{time period}}$$

∴ angular velocity of precession

$$\boxed{\frac{d\theta}{dt} = \phi \omega_0 \cos \omega_0 t}$$

→ when that $\cos \omega_0 t = 1$ then.

$$\boxed{\frac{d\theta}{dt} = \phi \omega_0}$$

→ the max angular velocity of precession

$$\omega_p = \phi \omega_0$$

From gyroscopic

: entrostic (1)

$$\text{bend of gyrosc} = I \omega \omega_p \sin \phi \cos \theta \rightarrow \text{entrostic will be}$$

$$= I \omega \cdot \omega_0 \text{ bend. now if } \omega_0 \text{ is constant}$$

$$= I \omega \phi \left(\frac{2\pi}{T-p} \right) \text{ bend. if } \omega_0 \text{ is constant}$$

$$\text{II large angular accel}^n = \phi \omega_0^2 \sin \omega_0 t.$$

$$\therefore \text{max angular accel}^n = \phi \omega_0^2.$$

3) Rolling :

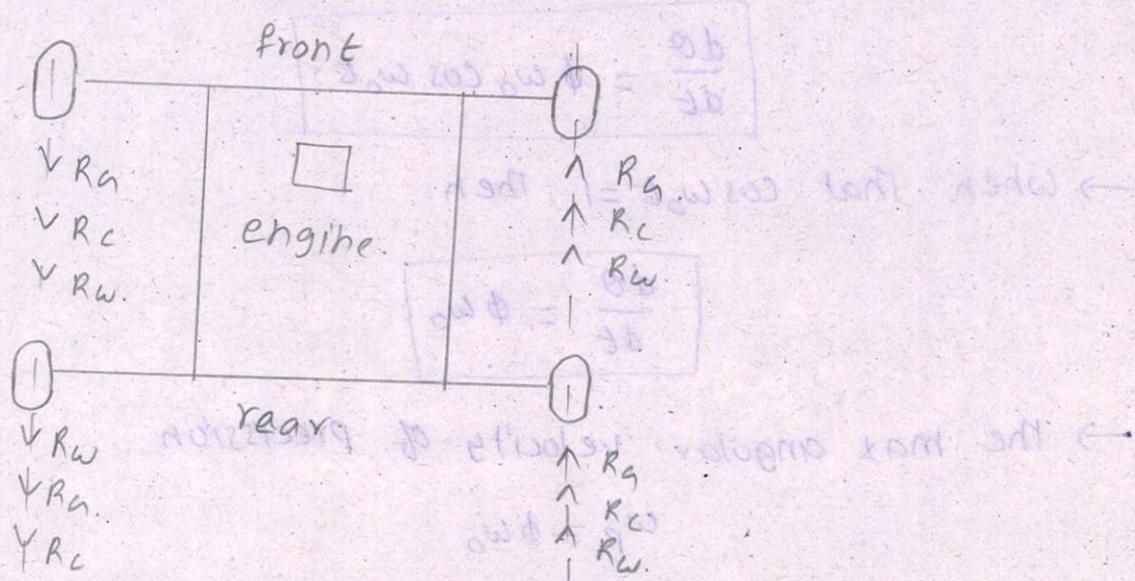
The axis of the rolling of the ship and the rotary are parallel,

There is no precession of the axis of spin and thus there is no gyroscopic effect.

→ Some were the effect of steaming, pitching and rolling can be observed when the plane of spin of rotating masses is horizontal but along the longitudinal axis of the vessel.

are when the axis - vertical.

Gyroscopic effect on four wheel (motor car).



→ The four wheel vehicle essential no wheel is repeated of the ground. When the vehicle turns a turn, the four wheel vehicle is having a mass of 'm' assumed that weight is divided among the four wheel.

• Weight of each wheel is

$$\frac{W}{4} = \frac{mg}{4} \text{ (Upward)}$$

Reaction on the down wheel is

$$\frac{W}{4} = \frac{mg}{4} \text{ (Downward)}$$

→ gyroscopic couple effect on the two ways.

i) Effect of couple.

2) Effect of centrifugal couple.

Effect on gyroscopic couple :-

The gyroscopic couple due to 4 wheel is

$$C_w = \omega_w \omega_p \cdot I_w \times 4$$

$$C_w = 4 \cdot I_w \omega_p \omega_w$$

The gyroscopic couple in the rotation of engine.

$$C_e = I_e g \omega_w \omega_p$$

$$\alpha = \frac{\omega_e}{\omega_w}$$

∴ The gyroscopic couple on the center of gravity

$$C_g = I_e + C_w$$

The force on the two outer wheel = $\frac{C_g}{w}$ (downwards).

The force on the two inner wheel = $\frac{C_h}{w}$ (upward).

The force on each of the outer wheel = $\frac{C_g}{2w}$ (downward).

The force on each of the inner wheel = $\frac{C_h}{2w}$ (upwards).

reaction of the ground on each outer wheel $R_o = \frac{C_g}{2w}$

effect on centrifugal couple :-

As a vehicle moves on curved path a centrifugal force also acts on the vehicle.

$$C_c = m r w^2 = m \cdot R \cdot \left(\frac{v}{R}\right)^2$$
$$= \frac{m v^2}{R}$$

∴ This force would tends to overturn the vehicle outward & out turning will be w .

$$C = m R w^2 \times h$$

$$C = m \frac{v^2}{R} \times h$$

→ vehicle reaction on each of the outer wheel

$$R_o = \frac{w}{4} + \frac{C_g}{2w} + \frac{C_e}{2w}$$

Effect on motor cycle :-

For each effect on gyroscopic couple in a body

$$C = \frac{V^2}{R \times r_w} [2I_w + GI_E] \cos\alpha$$

For each effect on centrifugal couple

$$C_c = \frac{V^2}{R} \left[\frac{2I_w + GF_E}{r_w} + m \cdot n \right] \cos\alpha$$

Applications of gyroscopic couple :-

1) gravity diffusions.

2) air & land vehicles.

3) ships, overcarfts.

4) initial compass.

Static and dynamic force analysis.

Static force analysis :-

→ if the compounds of the acceleration initial force procedure in these masses, however if the magnitude of this force are small compare come back to external applied loads, they can be neglected by analysing the mechanism such analysis is known as static force analysis.

dynamic force analysis :-

when the initial force effect due to the mass of the component is also considered called dynamic force analysis.

Static equilibrium :-

A body is in static equilibrium, if it remains in its state of rest in motion, the body is at rest in terms to remain at rest & its motion.

Therefore

- 1) The vector sum of the all the forces acting on the body is 0.
- 2) The vector sum of the all the moment about any arbitrary point is zero.
- 3) $\sum F = 0$ & $\sum r = 0$.

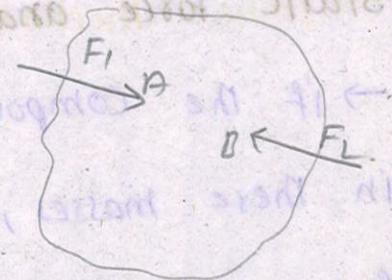
equilibrium of 2 or 3 force members:-

1) A member under two force will be equilibrium.

→ The force are same magnitude $F_1 = F_2$.

→ The forces acting on the same line.

→ The forces are in opposite direction.

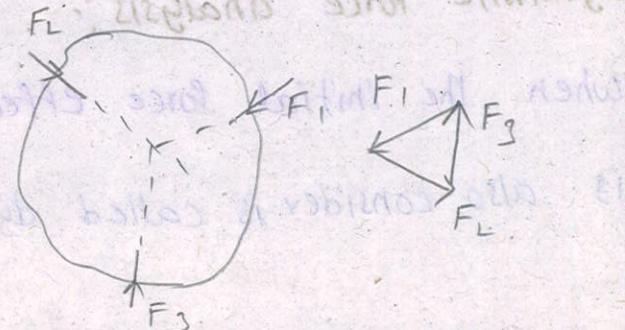


2) A member under 3 forced will be equilibrium

→ The resultant of force is 0 & $R = 0$

→ The line of the action of the force intersect at the point.

is known as point of concurrency.



member with 2 force and torque :-

A member under the action of 2 forces and applied torque will be equilibrium if

1) The force are equal in magnitude & in direction & opposite in signs.

2) The forces form a couple which is equal in opposite to the applied torque of F_L .

$$T = F_1 \times h.$$

$$T = F_L \times h.$$

→ The forces F_1 & F_L are acting

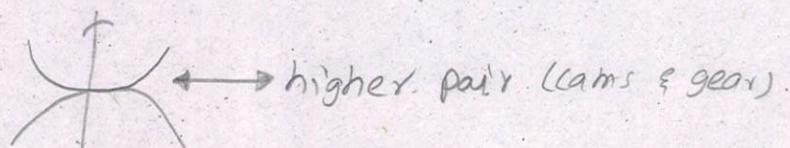
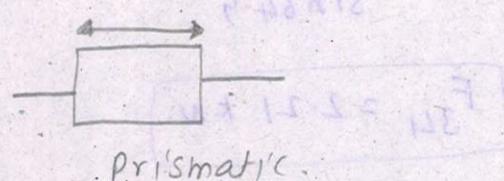
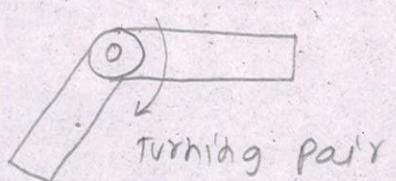
→ The torque acting on forces F_1 & F_L is clockwise direction then the forces counter.

Static force analysis of planer mechanism :-

→ Planer mechanism :-

All the relative motion of rigid bodies are in one plane.

→ In this analysis are usually do for this 4 bar mechanism for single crank mechanism



Static force analysis of single slider crank mechanism:

vector diagram :-

$$\rightarrow \sin 60^\circ = \frac{AC}{OA}$$

$$\Rightarrow AC = OA \times \sin 60^\circ \\ = 100 \times \sin 60^\circ$$

$$= 86.6 \text{ mm}$$

In $\triangle ACR$

$$\sin \beta = \frac{AC}{AB}$$

$$= \frac{86.6}{200}$$

$$= 0.43$$

$$\beta = \sin^{-1}(0.43)$$

$$\beta = 25.65^\circ$$

Forces :-

By Iami's theorem

$$\frac{2 \text{ kN}}{\sin 64.4^\circ} = \frac{F_{34}}{\sin 90^\circ}$$

$$F_{34} = \frac{2 \times \sin 90^\circ}{\sin 64.4^\circ}$$

$$F_{34} = 2.21 \text{ kN.}$$

$$\cos(60 - \beta) = \frac{OB}{AB}$$

$$T = AB \times \cos(60 - \beta)$$

$$= 200 \times \cos(60 - 25.4)$$

$$= 164.62 \text{ m.}$$

D'Alembert's principle :-

It states that initial force & couple and the external of force & torque on a body together give statical equilibrium. The inertia is property of matter by virtue of which a body resist any change in velocity.

$$\therefore F_i = -m F_g$$

where m = mass of body &

F_g = accelⁿ of center mass of the body.

'-' indicate sign that the force acting the opposite direction that of accelⁿ.

II^{law} The inertia couple resists any changes in the angular velocity.

$$\text{inertia couple } C_i = -I_g \alpha$$

where I_g = moment of inertia about an axis passing through center of mass & is \perp to the plane of rotation of the body.

$$\alpha = \text{angular accel}^n$$

equivalent offset inertia force

→ in the plane motion involving accelⁿ the inertia force act on the body, through its center of mass, whenever, if the body is acted on up a force such that resultant does not pass through the center of mass. A couple also act on the body. The linear displacement 'h' on the force from the center of mass is such that the torque produced is equal to inertia couple produced in the body.

$$T_i = c_i$$

$$T_i = F_i \times h$$

$$F_i \times h = c_i$$

$$\frac{h}{c_i} = \frac{-I_g \alpha}{F_i} = \frac{-m k^2}{m f_g} = \frac{m k^2}{m f_g} = \frac{k^2}{f_g}$$

Dynamic analysis of 4 link mechanism

→ Draw the velocity & accelⁿ diagram of the mechanism from the configuration diagram by usual methods

→ Determine the linear accelⁿ of the centers of masses of various links and also the angular accelⁿ of the links.

→ calculate the inertia force and couple from the reactions
 $f_i = -m f_g$, $\sum c_i = -I_g \alpha$.

→ replace F_i with equivalent offset to inertia force to take into account F_i & as well as c_i .

→ Assume the equivalent offset inertia force on links as static force and analyse the mechanism by any ff method.

The dimensions of a four-link mechanism are AB is 500 mm, BC is 660 mm and CD is 560 mm and AD is 1000 mm, the link AB has angular velocity of 10.5 rad/sec counter clockwise and angular retardation is 26 rad/sec at the instant angle of 60° with AD the fixed link, the mass of link BC & CD is 4.2 kg/m of length and link AD has a mass of 3.54 kg, the center of which lie at 200mm from A & moment of inertia of 885.00 kg/mm² neglecting gravity and friction less & determine the instant value of the drive torque required to be applied on AB to over come the inertia force.

Given data

The dimensions of

$$AB = 0.5m$$

$$BC = 0.66m$$

$$CD = 0.56m$$

$$AD = 1m.$$

Angular velocity of AB is $\omega = 10.5 \text{ rad/sec}$.

" retardation is $\alpha = 26 \text{ rad/sec}^2$.

$$\text{angle } \theta = 60^\circ$$

The mass of AB is $m_{ab} = 3.54 \text{ kg}$

$$m_{bc} = m_{cd} = 4.2 \text{ kg/m}$$

radius from center is $r = 200 \text{ mm}$

moment of inertia $I = 88500 \text{ kg-mm}^2$

→ from the velocity diagram

$$v_{ab} = \omega_{ab} \times AB = 10.5 \times 0.5 = 5.25 \text{ m/sec}$$

$$v_{bc} = 3.4 \text{ m/sec}$$

$$v_{dc} = 3.9 \text{ m/sec}$$

→ from the accelⁿ diagram

$$\text{The accel}^n \text{ of AB at point C is } f_{ab} = 55.1 \text{ m/sec}^2.$$

$$f_{bc} = \frac{3.4^2}{0.66} = 17.51 \text{ m/sec}^2$$

$$f_{dc} = \frac{3.9}{0.56} = 27.1 \text{ m/sec}^2$$

→ The mass of the link

$$m_{ab} = 3.54 \text{ kg}$$

$$m_{bc} = 2.77 \text{ kg}$$

$$m_{dc} = 2.35 \text{ kg}$$

→ locate the 1st point in accelⁿ diagram

$$\therefore f_{g_1} = 22.6 \text{ m/sec}^2$$

$$f_{g_2} = 52.0 \text{ m/sec}^2$$

$$f_{g_3} = 25.7 \text{ m/sec}^2$$

The inertia of couple and angular accelⁿ of the links

$$\alpha_2 = 26 \text{ rad/sec}^2$$

$$\alpha_3 = 39.3 \text{ rad/sec}^2$$

$$\alpha_4 = 46.4 \text{ rad/sec}^2$$

We know that

$$h = \frac{C_i}{F_i}$$
$$= \frac{k_i^2 \tau}{f g}$$

$$\text{where } k_i^2 = \frac{88500}{3.54} = 25000 \text{ mm}^2$$

$$k_2^2 = \frac{660}{12} = 36,300 \text{ mm}^2$$

$$k_3^2 = \frac{500}{12} = 26103.3 \text{ mm}^2$$

$$h_2 = \frac{k_2^2 \tau}{f g_2} = \frac{25000 \times 26}{22600} = 28.76 \text{ mm}$$

$$h_3 = \frac{36,300 \times 39.39}{52000} = 22.42 \text{ mm}$$

$$h_4 = \frac{26103.3 \times 46.42}{25,708} = 47.2 \text{ mm}$$

